star, the fainter may its absolute luminosity be. One of Russell's diagrams, in which the absolute magnitudes are referred to a distance of 10 parsecs, is reproduced in Fig. 6. In this diagram the small dots represent individual stars, the large circles mean values for bright stars of small proper-motion and parallax. It will be seen that the general distribution of the dots is along two lines inclined at an acute angle and intersecting at type B; that this distribution is not the result of the selection of stars for parallax determination on the ground of brightness or size of proper-motion was conclusively shown by Russell. It will also be seen that for the red stars there is a complete separation between the two classes, so that a very red star is intrinsically either very bright or very faint. These facts have given rise to the "giant" and "dwarf" hypothesis, and have led to a recasting within the last few years of the ideas as to stellar evolution which were formerly generally accepted.

The following results emerge from Russell's investigation: (1) Stars of all types occur brighter than zero absolute magnitudes,[2] and mostly between 0 and −2M—say, about 150 times the luminosity of the sun. These are called "giant" stars. (2) There are no B-type stars, and very few A-type stars, fainter than zero absolute magnitude, or, in other words, all the white stars are intrinsically very bright. (3) All the faint stars, less than, say, 1/50 the luminosity of the sun, are red and of types K and M. These are called "dwarfs," and comprise all the near stars of large proper-motion. (4) In the intermediate classes, F and G, there is no separation between the giants and the dwarfs. Our sun (5·0M) is a typical G-type star. In view of these remarks, it is obvious that no precise meaning attaches to a statement such as "The average absolute magnitude of all stars is +2·7M."

Shapley's work on the magnitudes of stars in clusters, combined with his determination of the distances of clusters, has shown that the giant stars in clusters, which are the only ones sufficiently bright to appear on the photographs, are of about the same magnitude as the giant stars in our more immediate neighbourhood. Two further points of interest emerge from the investigation: one is that in all the clusters examined in detail the intrinsically brightest giant stars are red stars; this may also be true for the stars near the sun, although the determinations of their absolute magnitude are probably not sufficiently accurate to show it; the other point is the apparent importance of an absolute magnitude of about −0·2M. Shapley finds that all Cepheid variables and cluster variables exceed this brightness; moreover, in the luminosity curve which connects the number of stars of any given absolute magnitude with the magnitude, there is a maximum in the curve corresponding to the same magnitude. In Shapley's opinion, this magnitude—corresponding to a luminosity of about 100 times that of the sun—indicates a critical stage in stellar evolution, and, in all probability, is of significance in the theory of a gaseous star. It seems, in fact, probable that by the new methods recently discovered for estimating great distances, combined with the advantages afforded by the large reflecting telescopes at Mount Wilson, we may learn more about absolute magnitudes from a study of clusters at distances corresponding to parallaxes of the order of 0″·00005 than from the study of the stars which immediately surround us.

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[2] The unit of absolute magnitude used here is that which corresponds to a parallax of one-tenth of a second of arc.

Dynamics of Golf Balls.

THE physical principles underlying the flight of a golf ball were clearly laid down by the late Prof. Tait between the years 1890 and 1896.[1] In view of the present agitation over the standardising of the golf ball, it may be of advantage to reconsider some of the problems attacked by Tait and largely solved by him. The investigation led him into a series of researches on impact so as to obtain data for measuring the resilience of the material of which golf balls were then made. Also, by means of a specially constructed ballistic pendulum, measurements were made of the speed of a golf ball impinging on the pendulum placed at a distance of about 6 ft. from the tee. By attaching a tape to the ball, Tait was able to obtain direct measurements of the amount of underspin communicated to the ball at the instant of striking it. Outside observations were also made of the heights of the trajectories of well-driven balls, and of the ranges and times of flight. All these data were skilfully introduced into the mathematical discussion of the form of the trajectory, a problem so difficult as to be capable of solution only by approximate methods. This was done before the days of the rubber-cored ball, and the steady improvement in the manufacture of the golf ball has enabled even very ordinary players to exult in lengths of drive which in Tait's days were beyond the powers of the mightiest exponents.

What Tait established beyond all controversy was that the range of the trajectory of a properly driven ball depended as much upon the underspin as upon the speed of projection. The combined effect of the linear speed and the rotation about a horizontal axis brought into play a force perpendicular to the direction of motion of the ball. Tait gave sound reasons for regarding this force as being proportional to the product of the velocity and the spin. Thus, although the possibility of a long trajectory depends primarily upon the velocity of projection, the range actually attained in any particular case will be governed by the amount of underspin communicated to the ball. If this is too great, the ball will rise too high, and the range will be correspondingly diminished. If the underspin is too small, gravity will predominate and pull the ball more quickly down to earth, with resulting diminution of range. For every velocity of projection of the ball which leaves the tee in a horizontal direction there will be a best value of underspin enabling it to attain the greatest range in still air. The art of the golfer is to manipulate his club so as to give this necessary amount of underspin.

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[1] "On the Path of a Rotating Spherical Projectile." Trans. R.S.E., 1893 and 1896; "Some Points in the Physics of Golf," NATURE, vols. xlii., xliv., and xlviii.; "Long Driving," *Badminton Magazine*, 1896.

It is probably not realised by many efficient golfers how much this underspin may be varied by small changes in the position of the line of stroke of the club as it hits the ball. Let us take Tait's maximum estimate of 120 revolutions or about 750 radians per second as the value of the underspin, and consider how far below the centre of mass of the ball the line of impulse must be so as to send the ball off with this spin and a speed of 300 ft. per second. The ball is supposed to be hit horizontally off the tee without any reactionary upward or backward impulse acting on it. The distance x below the centre of mass at which the line of impulse must act so as to give this combination of linear speed and spin has the value $x=k^2\omega/v$, where k is the radius of gyration of the ball, and v and ω are the speed and spin respectively. With $k^2=0{\cdot}276$ in.^{2}, and $v=3600$ in./sec., we find $x=0{\cdot}054$ in. A variation of one-hundredth of an inch in this value will change the spin by nearly 20 per cent. Such variations may easily be effected by very slight changes in the lie of the club head.

With a given ball the velocity of projection and the spin are the only factors which are under the control of the player. Once the short time of impact between the club face and the ball is completed, nothing the player can do can influence the flight of the ball. Thereafter all is determined by the combined influence of gravity and the air.

So far as the player is concerned, the velocity of projection depends mainly upon the velocity of the club at the moment it strikes the ball. The weight behind the stroke no doubt has a secondary influence, but the great thing is the *swiftness* of the stroke. For this reason experience has evolved a weight of club which is found most serviceable for the strength of the average man. In an ordinary driver, weighing (say) 1 lb., probably one-third of the weight is in the club head; and if we were to think of the problem as one of simple impact between two masses of which one is at rest, we might work out the relative velocities of club and ball after impact for an assumed value of the coefficient of restitution. But the conditions of the problem are not so simple. The player, by the swing of his body and arms and well-timed effective wrist play, not only imparts a rapid acceleration to the club head up to the moment of impact, but in all probability imparts, unconsciously, perhaps, but none the less effectively, an acceleration *during the time of impact,* short though that be. In spite of the back impulse on the club as it is striking the ball, its velocity is kept up by the unconscious knack of the player. The relative velocity with which the ball leaves the club is e times the momentary velocity of the club, where e is the coefficient of restitution, and hence the velocity of projection will be $(1+e)$ times the velocity with which the club is moving at the instant club and ball separate.

Outside the factors over which the player has some control, the most important is the resilience of the ball, and the steady improvement in this quality is, of course, at the root of the great increase in lengths of drive. It was this question of resilience which, indeed, started Tait on his investigations on impact. The apparatus designed by him for the purpose was nicknamed the "guillotine." It consisted fundamentally of a weight which, guided by upright parallel slots, was dropped on the ball or other body the elastic properties of which were under investigation. The heights reached by the weight after successive rebounds were recorded automatically on a rotating disc 2⅓ ft. in diameter. From the record all the facts of the impact could be derived more or less directly, such as the compression of the ball, the duration of the impact, and the value of e, the coefficient of restitution. The weight was made of wood, but its lower face could be, when required, shod with an iron plate.

The recording part of the apparatus has long been dismantled, but the "guillotine" part is still serviceable. In order to compare the values of e for modern golf balls with the values obtained thirty years ago by Tait, impact experiments were recently carried out on sixteen balls of recognised merit—namely, various types of Avon ball, Challenger, Clincher Cross, Dunlop, Silver King, and Spalding. Thanks are due to the Avon India Rubber Co., Ltd., J. P. Cochran, Ltd., North British Rubber Co., Ltd., Dunlop Rubber Co., Ltd., and A. G. Spalding and Bros., Ltd., for their kindness in supplying specimens of balls of the best quality. With the exception of five, all were of greater diameter than the new standard minimum, and only two exceeded the maximum standard weight. Their specific gravities varied from 1·07 to 1·29. On each of these balls the weight of 4·75 lb. was allowed to fall from a height of 9 ft., and the height of the first rebound was noted. The square root of the ratio of these heights gave an approximate value for e, and this was corrected by comparison with Tait's results, which showed that under the conditions of the experiment the ratio of the speeds immediately after and immediately before the impact was greater than the estimate from the corresponding heights by about one-ninth. The average value of e for the sixteen balls mentioned was 0·72, the lowest being 0·71, and the highest 0·75. Tait obtained for the balls he experimented with the value 0·66. He estimated 300 ft. per second as a fairly probable value for the velocity of projection. On the assumptions indicated above, this would imply a velocity of projection of 311 ft./sec. for the ball with coefficient of restitution equal to 0·72.

This does not seem to indicate any very marked superiority in the modern ball—at least, it cannot explain the greatly increased length of drive attainable in these days. The reason is to be sought in the fact that the conditions of constraint under which the impact experiments are made are essentially different from those under which a golf ball is compressed and distorted as it is propelled freely in its flight. Everyone knows that the high resilience of the rubber-cored ball is derived from the fine rubber thread which is wound on under considerable tension. Before the outer covering is put on, these balls, when dropped from a height of 6 ft. or 7 ft., rebound from stone or metal to a height which indicates that the coefficient of restitution exceeds 0·8. When we consider the manner in which this complex of tightly wound rubber resists any sudden distortion produced by a short-lived blow, we shall probably be prepared to admit that such an elastic complex will resist compression more powerfully than an equal sized ball of vulcanised india-rubber, which Tait found to have a coefficient of restitution greater than 0·8. Any impulse brought to bear upon one part of the rubber-wound ball will produce in every strand of the rubber thread an immediate tightening with corresponding resistance to change of shape.

Let us suppose, then, that, under these conditions, the coefficient of restitution approaches the value unity, say 0·95. If the old gutty with coefficient of restitution 0·66 was propelled with an initial velocity of 300 ft./sec., then this ball, with coefficient of restitution 0·95, will be projected with initial speed of 356 ft./sec. This by itself will not account for an increase of 70 or 80 yards in the length of drive, for, as pointed out by Tait, a greater initial speed means a greater air resistance; and (other things being the same) to add 83 yards to the length of a drive means double the velocity at start. But here, again, we may invoke the influence of the underspin. As already stated, there is for every velocity of projection a definite value of underspin which will enable a given ball to travel its farthest range. Since the upward force produced by the combined action of the linear velocity and spin depends on both these factors, an increased velocity of projection will have to be associated with an increased rate of spin if its greatest range is to be attained. The problem is one which would well repay working out in detail.

If great length of drive is a desideratum in the game of golf, then undoubtedly the "floater" must give way to the heavy ball. This is a simple illustration of the well-known law of atmospheric resistance, the effect of which upon a sphere passing through the air is directly proportional to the surface, and inversely proportional to the mass. The accurate driver finds by experience that a heavy small ball travels farthest through the air. For example, if we make a floater of density unity and of the maximum weight, its diameter will be 1·75 in. The retarding effect of the resistance of the air on this floater will be 17 per cent. greater than the retardation experienced by the new standard ball of minimum size and maximum weight. Again, if we make a floater of the minimum size, its weight will be only 1·28 oz., and it will experience a retardation due to atmospheric resistance which will be nearly 27 per cent. greater than that experienced by the standard "minimax," to use a word introduced long ago by Kelvin in a different connection. The "minimax" itself experiences slightly less atmospheric resistance than most of the balls mentioned above, being excelled in this respect only by Dunlop 31, Spalding Midget, small Avon de Luxe, and Silver King; but the difference never reaches 2 per cent. It is therefore not surprising that long driving is also attainable with the standard "minimax" ball.

A reference has been made to the radius of gyration of a golf ball as a factor influencing the amount of spin communicated to the ball. The square of the radius of gyration of a uniform sphere is $\frac{2}{5}r^2$, where r is the radius of the sphere. By means of oscillatory experiments, in which the golf ball was supported by a ring-shaped disc hung by a tri-filar suspension from three fixed points, the moments of inertia and radii of gyration of all the golf balls used were determined to an accuracy of about 1 per cent. The moments of inertia expressed in grams and centimetres varied from 86 for the Large Heavy Avon to 66 for the Standard Clincher Cross, and yet the mass of the latter was slightly the greater, being 45·4 grams (1·60 oz.), as compared with 44·6 (1·57 oz.). This great difference in the moments of inertia depends on the distribution of matter within the ball. The value of k^2 for the larger balls was practically the same as the value $\frac{2}{5}r^2$ for the uniform sphere of equal size; but in the case of the small balls k^2 was markedly less than $\frac{2}{5}r^2$, being in some cases as much as 8 per cent. smaller. The reason is that the small balls have a very dense core. It is obvious that with the larger moment of inertia a greater moment of impulse must be given to obtain the same spin. But this is automatically effected, since with the same club the larger ball is struck along a lower line relative to the centre of mass, so that the moment of the impulse is of necessity greater. During the flight of the ball the larger moment of inertia will enable the ball to conserve its spin the better, which will probably have a beneficial effect on the range or carry.

It appears, then, that the length of drive attainable depends on several factors, and of these the most effective are the resilience of the ball and the underspin given at the instant of impact. To drive a long ball is one of the delights of golf, and the ball which travels farthest will be the favourite. By almost all young and vigorous players the floater, because of its lightness, is regarded unfavourably. It lacks, comparatively speaking, steadiness in the air and accuracy on the greens, and cannot possibly be driven so far. It is little wonder that the heavy ball has ousted it in all serious play.

It is not the purpose of this article to touch on the question of standardisation of the golf ball. Its aim is to discuss the physical principles which govern the flight of the ball through the air. But the physiological and psychological powers or weaknesses of the player are of equal importance. There is a limit to the weight of club which can be most efficiently used by the average man, and there must also be a limit to the weight of the ball. From the point of view of atmospheric resistance, the ratio of the surface to the weight must be kept as low as possible; but too small a surface will diminish the lifting power of the underspin, just as too large a weight will cut down the velocity of projection. The one quality which must be as perfect as possible is the resilience of the material; but no ball can have a higher coefficient of restitution than unity, and therefore no ball can start on its flight with a velocity greater than twice that of the club head at the instant of impact. Physical and physiological considerations necessarily fix a limit to the range of flight attainable, and probably that limit is now being approximated to. Which, then, is simpler—to standardise or to re-arrange our golf courses?

C. G. K.

Nature in a Himalayan Valley.[1]

By LT.-COL. J. H. TULL WALSH.

WE have here the notes made by an officer of the Indian Medical Service in the Hazara valley of the foot-hills, during the years 1914–16. These observations are wide in their range, and were, no doubt, a relief to more serious work. The author is an amateur naturalist, far from works of reference and museum specimens, and the opinions are strictly personal. No man can possess full knowledge in all the branches of science alluded to—for there is compilation as well as observation in this book—but Capt. Hingston has acknowledged his borrowings. The ordinary lover of Nature, who likes a pleasantly written account of geology and animal life in an area not well known to many, will enjoy this book, ignoring opinions with which he may not agree, and errors which the technical naturalist would claim as serious. The general features of the Hazara valley are shown on the map facing p. 4. It is a "slender wedge of British soil" about 120 miles long, its width varying from 56 miles at the base of the wedge to 15 miles at the apex. "To the south its foot-hills sink into the plains of the Punjab; to the north it rises into massive peaks 17,000 ft. in height that blend with the still loftier summits of western Kashmir."

The first five chapters are devoted to ants, harvesting ants, a species placed in the genus Myrmecocystus, and others. Habits, etc., are freely discussed, and a great deal is written concerning instinct. The author asks too much from instinct, and "folly" (p. 41) is scarcely the correct word to apply to mistakes which are not provided for among instincts inherited by insects. Two plates (facing pp. 13 and 60) are given of certain ants commonly found in the Hazara valley. While on the subject of plates, we think it would have been better to give them numbers. The illustrations themselves are excellent, as our examples prove.

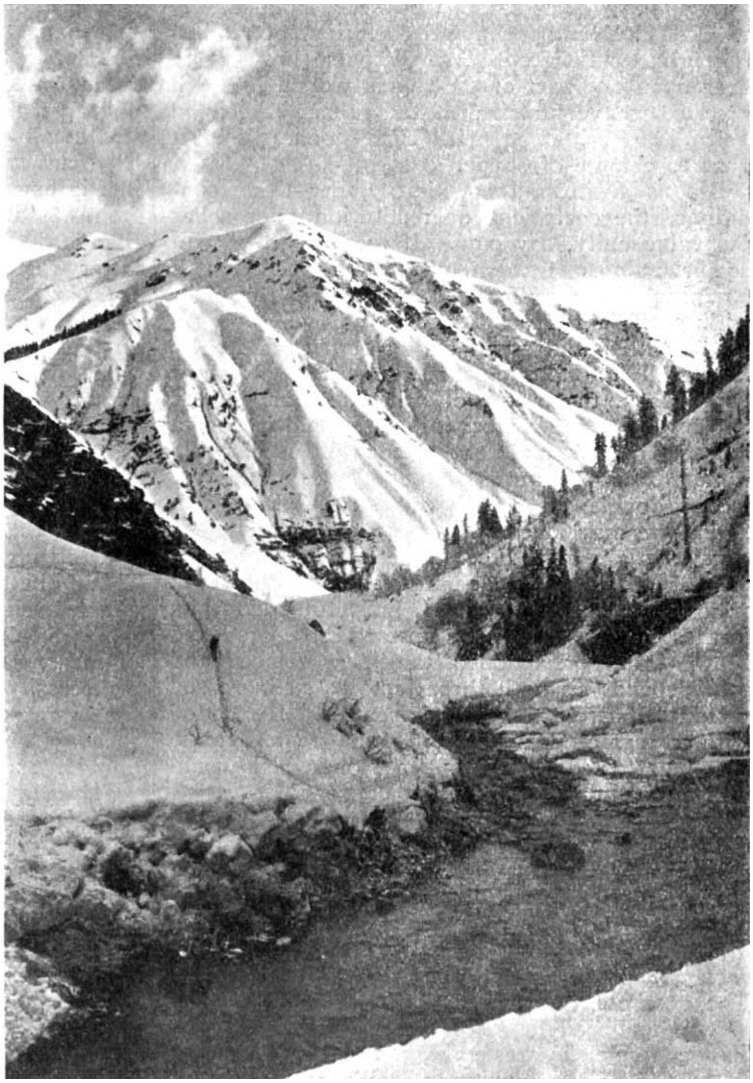

FIG. 1.—In the Himalaya. From "A Naturalist in Himalaya."

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[1] "A Naturalist in Himalaya." By R. W. G. Hingston. Pp. xii+300 +plates. (London: H. F. and G. Witherby, 1920.) 18s. net.